ENERGY/COLOR FLOW

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OUTLINE

- Motivation
- Rapidity gap process
- Factorization and refactorization
- Color mixing matrices
- Resummation
- Gap fraction
- Summary

MOTIVATION

- The role of perturbative QCD
  - Tevatron
    - QCD was a theory to be tested.
      - Discovery of top quarks
  - LHC
    - Precision
    - A tool to search for physics BSM

MOTIVATION

- The role of perturbative QCD
  - Tevatron
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    - Precision
    - A tool to search for physics BSM
    - A tool to extract properties in BSM

MOTIVATION – BASIC PROPERTIES IN BSM

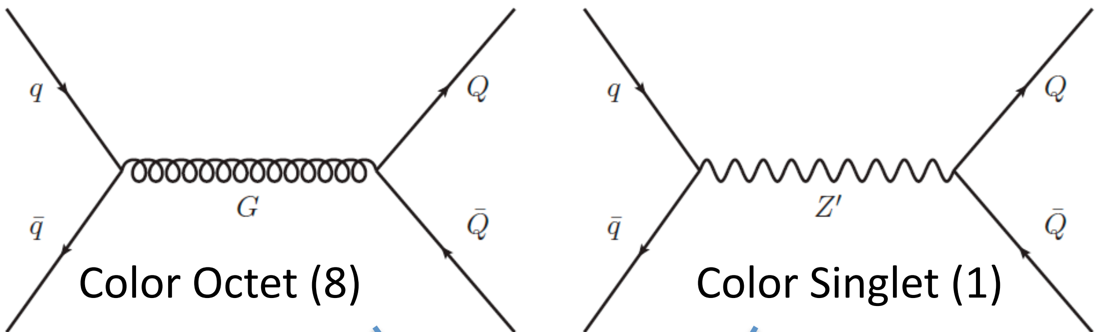
- There are many proposals for the extension of the SM.
  - The most common feature of the models is that they include new heavy particles.
  - They decay into the SM particles.
    - KK gauge boson -> a quark or lepton pair
      - [Arkani-Hamed,Dimopoulos, Dvali,1998], [Randall, Sundrum, 1999]
    - Z' -> HZ
- The detailed analysis of products of the resonance decay allows one to study the properties in new theory.
- Yes, but how?

MOTIVATION – SPECIFIC EXAMPLES WITH FOLLOWING CONSIDERATIONS

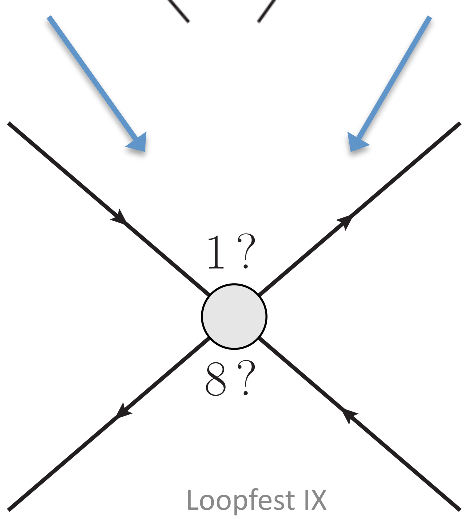
1) In many scenarios, heavy particles are created with O(TeV) mass, and decay into the SM particles.
2) The resonances are coupled strongly to heavy partons, compared to their couplings to light partons and leptons.
3) Let's assume that we succeed in distinguishing resonances from background events
   - Invariant mass distributions and many other techniques

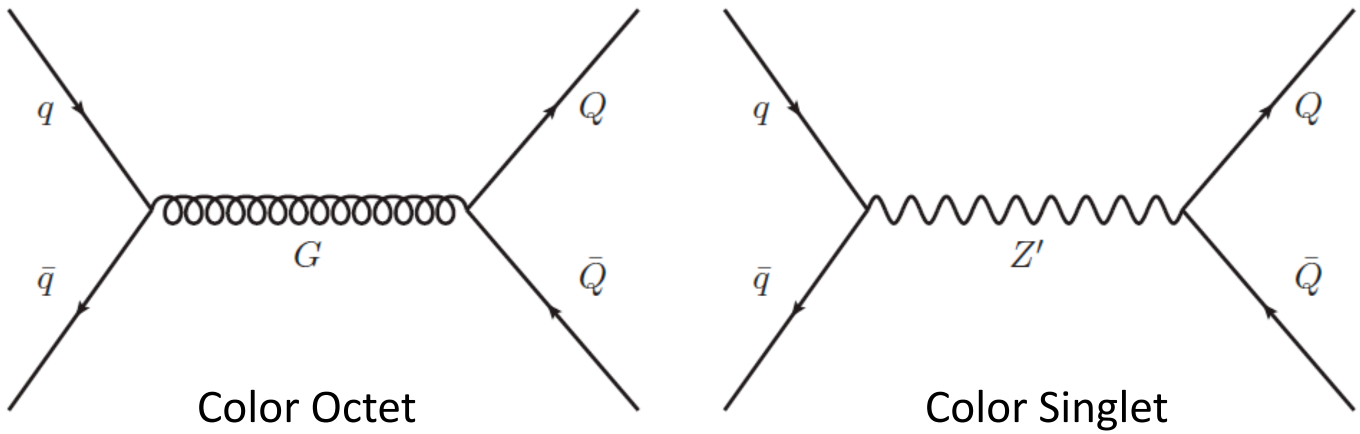
- What is next?
  - The SM gauge content of resonance particles
- Based on the previous considerations, we guess two possible resonance processes.

- What is the next?
  - The SM gauge content of resonance particles
- Based on the previous considerations, we guess two possible resonance processes. (s-channel due to strong couplings to heavy quarks)



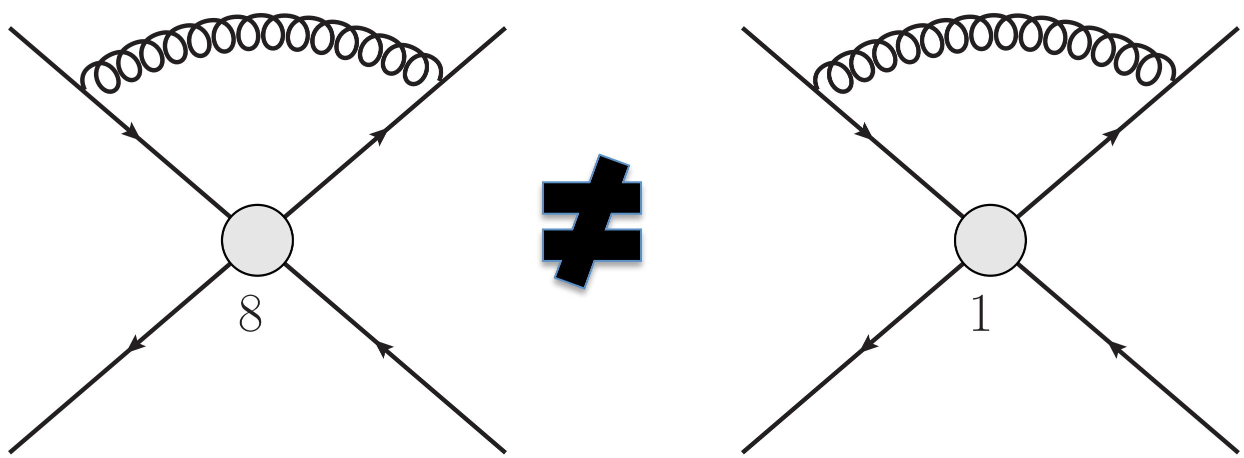
Goal :





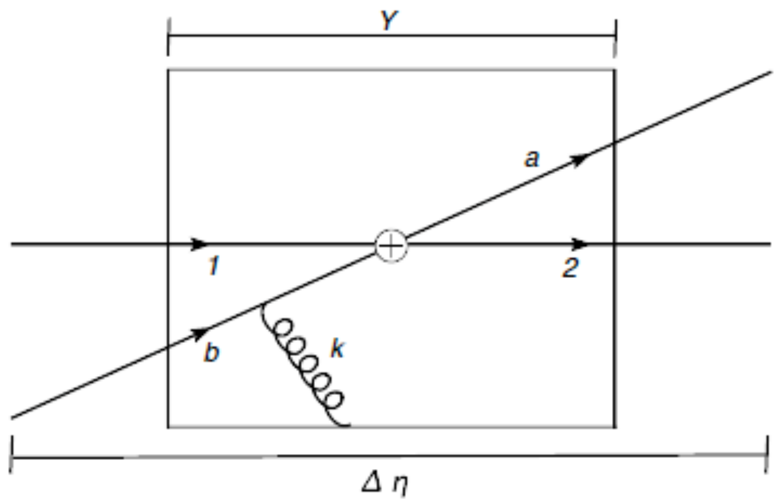
- Observation
  - All external particles are SM SU(3) particles.
  - Color can flow only through octet resonances (G).
- What is the physical quantity distinguishing one from another? And how can we calculate it?

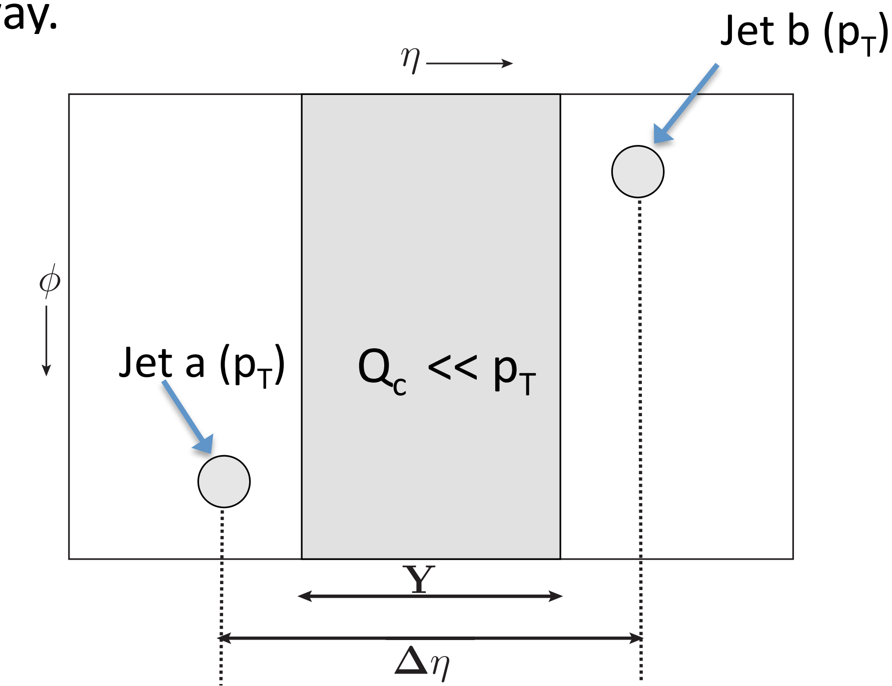
- The difference induced by different color flow in processes begins to appear at higher orders
  - Ex. We measure energy (Q_c) of gluon emissions into a certain area of a detector



- The different patterns of radiation may appear, depending on the SU(3) gauge content of resonances.

- How can we quantify the amount of radiation?
  - Rapidity gap processes [Oderda, Sterman (1998)], [Dasgupta, Salam (2001)], [Appleby , Seymour (2003)]
    - Note : This is not the only way.





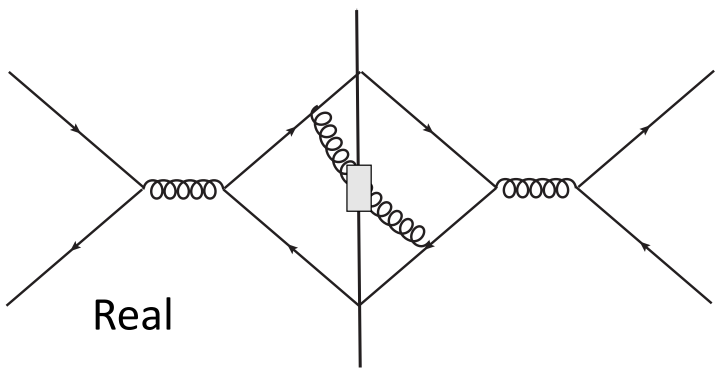
- We require to tag inclusively a pair of jets or heavy quarks and measure energy in a systematic central region, spanning rapidity Y.

- Our candidate quantity - inclusive top pair production cross-sections with energy in the central region
  - equal to Q_c $\quad \dfrac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta\, dQ_c}(M, m_t, Q_c, \mu_F, \Delta\eta, Y, \alpha_s(\mu_F))$
  - or up to energy flow Q_0

$$\frac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta}(M, m_t, Q_0, \mu_F, \Delta\eta, Y, \alpha_s(\mu_F)) = \int_0^{Q_0} \frac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta\, dQ_c}(M, m_t, Q_c, \mu_F, \Delta\eta, Y, \alpha_s(\mu_F))$$

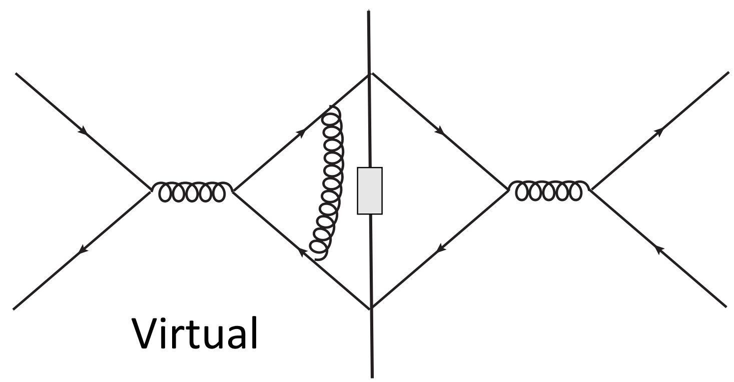
- When the latter is divided by total cross section, the ratio can be interpreted as a probability of a top pair production with radiation into the gap up to energy flow Q_0.

- Compare these probabilities, called "gap fractions", for color singlet and octet resonances.
- How can we calculate a gap fraction ?
  - Partonic cross section for each resonance process at fixed order.
  - MC simulations



Real

$$= \int_{\Omega_{\rm gap}} d\,\mathrm{PS}\,\omega_R$$



Virtual

$$= \int_{\Omega} d\,\mathrm{PS}\,\omega_V$$

$$\int_0^{Q_0} \frac{dk_0}{k_0} \int d\,\Omega_{\rm gap}\,(c_1\omega_R) - \int_0^{\infty\to\mu} \frac{dk_0}{k_0} \int d\,\Omega\,(c_2\omega_V) = C(Y)\ln\left(\frac{\mu}{Q_0}\right)$$

– However, we can do better than this by using factorization and evolution equations
  - Each loop produces a single log due to soft gluon exchange. [PRD17:2773,2789, Sterman (1978)]
  - Resummation; the sum of all leading enhancements along with associated color flow to all orders
– We measure observable from hadronic collisions. Thus we need a connection between partonic and hadronic cross sections.
  - Factorization [Adv.Ser.Dir.HEP, Collins, Soper and Sterman (1985)]

$$\frac{d\sigma_{AB}}{d\Delta\eta\, dQ_c} = \sum_{f_1,f_2} \int dx_1 dx_2\, \phi_{f_1/A}(x_1,\mu_F)\phi_{f_2/B}(x_2,\mu_F) \frac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta\, dQ_c}$$

REFACTORIZATION

- In general, we can perform a further factorization on the partonic rapidity gap cross section

$$\frac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta}(M, m_Q, Q_0, \mu_F, \Delta\eta, Y, \alpha_s(\mu_F)) \;=\; \sum_{L,I} H^{(\mathrm{f})}_{IL}(M, m_Q, \mu_F, \mu, \Delta\eta, \alpha_s(\mu)) \times S^{(\mathrm{f})}_{LI}\left(\Delta\eta, Y, \frac{Q_0}{\mu}, \alpha_s(\mu), m_Q\right)$$

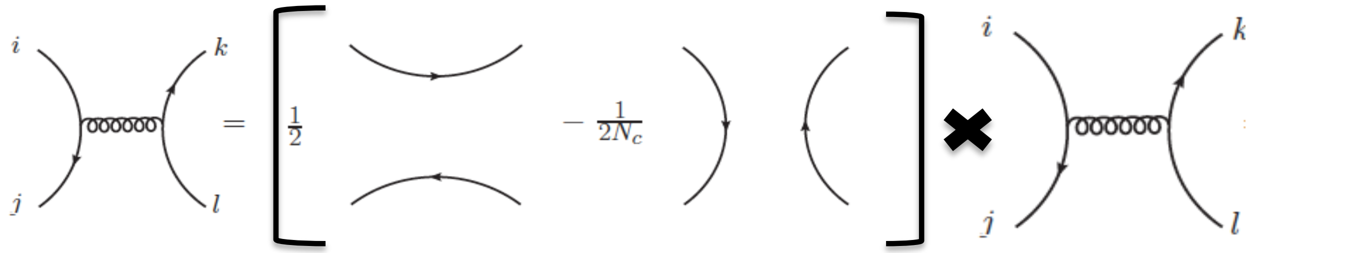
  - H represents physics at the large momentum scale
    - the mechanism of a heavy particle resonance is contained in H
  - S describes the infrared behavior of color exchange starting at short distance, and represents the radiation of soft gluons up to the scale Q_0.
  - I or L indicates some basis of color tensors linking external partons.

COLOR MIXING MATRIX

- How can we describe color flow ?
  - In fixed order calculations, we take amplitudes squared.
    - It is difficult to track color flow.
    - It is nontrivial to imagine color flow at higher orders.
  - Our goal : Describe color flow in the space of color tensors under exchange gluons
    [Sen (1983),Botts, Sterman (1989), Kidonakis, Orderda , Sterman(1998)]
    - Note: this is a key for resummation

- Amplitudes for our processes are four index tensors.
- Let's decouple color tensors from diagrams.
  - Use the color identity

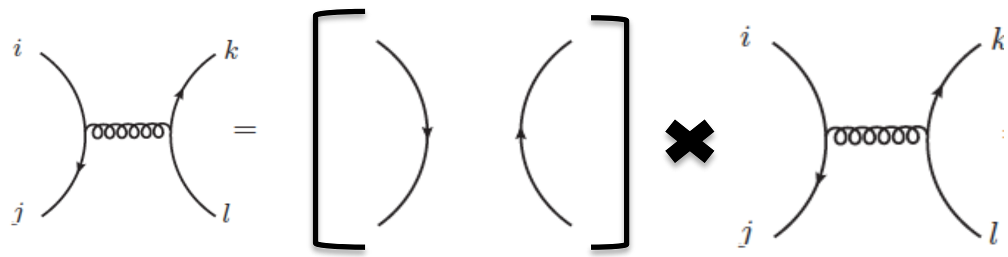
$$(T_F^a)_{ji}(T_F^a)_{kl} = \frac{1}{2}\left(\delta_{ki}\delta_{jl} - \frac{1}{N_c}\delta_{ji}\delta_{kl}\right)$$



Octet (G)

Color tensor $(C_2)_{ijkl}$

Diagram (kin. Only)



Singlet (Z')

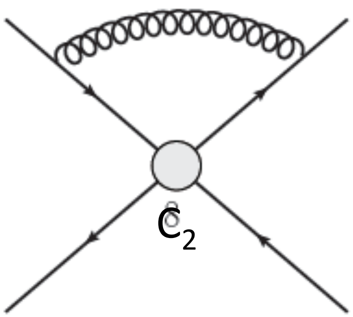
Color tensor $(C_1)_{ijkl}$

Diagram (kin. Only)

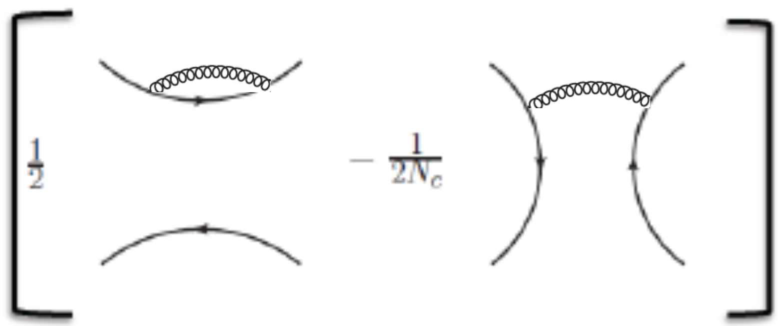
Resonance amplitude in color spaces -> hard function

COLOR MIXING MATRIX

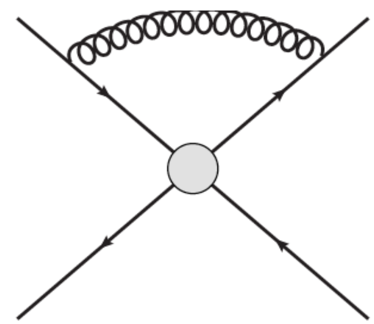
- Let's study how exchange of a gluon mixes the color tensors
  - If color tensor basis c_1 and c_2 span the entire color space, the result of color exchanges would be a linear function of two basis tensors.



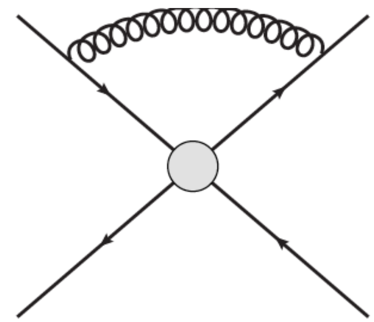
$=$



$\times$

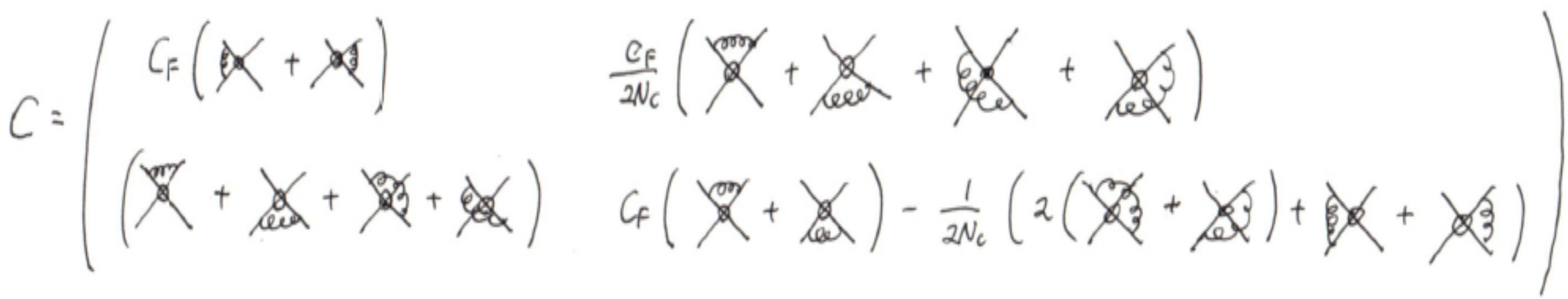


$$= \left[-C_F/(2N_C)(C_1)_{ijkl} \; - \; (CF-1/2N^2_C)\;(C_2)_{ijkl} \right] \times$$



COLOR MIXING MATRIX

- At one loop order, the color mixing matrix is given by [Berger, Kucs, Sterman (2001)]



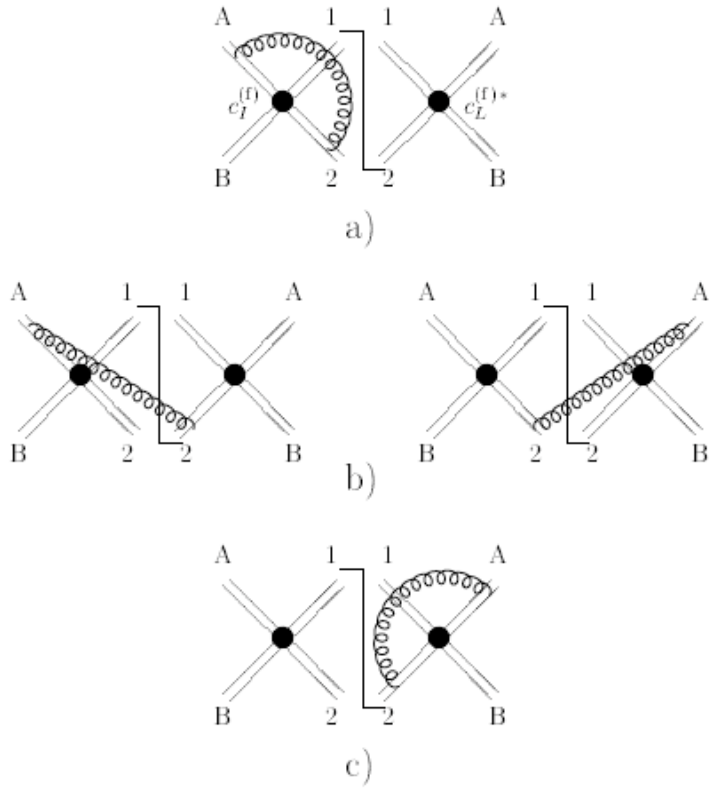
RESUMMATION

- Now let's try to sum leading logarithms at each order to all orders [Oderda, Sterman (1998), Berger, Kucs, Sterman (2001)]

$$\frac{d\hat{\sigma}^{(\mathrm{f})}}{d\Delta\eta}(M, m_Q, Q_0, \mu_F, \Delta\eta, Y, \alpha_s(\mu_F)) = \sum_{L,I} H_{IL}^{(\mathrm{f})}(M, m_Q, \mu_F, \mu, \Delta\eta, \alpha_s(\mu)) \times S_{LI}^{(\mathrm{f})}\left(\Delta\eta, Y, \frac{Q_0}{\mu}, \alpha_s(\mu), m_Q\right)$$

- Independence of the refactorization scale μ
  -> Evolution equations in μ

$$\mu \frac{d}{d\mu} S_{LI} = -(\Gamma_S)^{\dagger}_{LJ} S_{JI} - S_{LI}(\Gamma_S)_{JI}$$



- Eikonal calculation
  - Effectively captures only divergences
  - Scaleless integrals (UV+IR=0)

$$\gamma_{S\,(ij)}^{(1)} = \frac{\omega_{(ij)}(-2\varepsilon)}{(\alpha_s/\pi)}$$

$$\omega_{(ij)} \;=\; \omega_{V\,(ij)} + \omega_{R(ij)}$$

$$\omega_{(ij)} \;=\; \left(\frac{\alpha_s}{\pi}\right)\left(-\,\delta_i\delta_j\Delta_i\Delta_j\frac{1}{2}\frac{1}{2\varepsilon}\int\frac{dyd\phi}{2\pi}\Theta(\vec{k})\Omega_{ij} + \delta_i\delta_j\Delta_i\Delta_j\frac{i\pi}{2\varepsilon}\frac{(1-\delta_i\delta_j)}{2}\right)$$

$$\Omega_{ij} = \frac{(p_i\cdot p_j)k_T^2}{(p_i\cdot k)(p_j\cdot k)}$$

RESUMMATION

- Diagonalize color bases :

$$
\begin{aligned}
(\Gamma_S^{(\mathrm{f})}(\Delta\eta, Y, \rho))_{\gamma\beta} &\equiv \lambda_\beta^{(\mathrm{f})}(\Delta\eta, Y, \rho)\delta_{\gamma\beta} \\
&= (R^{(\mathrm{f})})_{\gamma I}\,(\Gamma_S^{(\mathrm{f})}(\Delta\eta, Y, \rho))_{IJ}\,(R^{(\mathrm{f})^{-1}})_{J\beta}
\end{aligned}
$$

- Evolution equation becomes

$$
\mu\frac{d}{d\mu}S_{\gamma\beta} = -(\lambda_\gamma^* + \lambda_\beta)S_{\gamma\beta}
$$

- Solution to this equation is

$$
S_{\gamma\beta}^{(\mathrm{f})}\left(\Delta\eta, Y, \frac{Q_0}{\mu}, \alpha_s(\mu), \rho\right) = S_{\gamma\beta}^{(\mathrm{f})}(\Delta\eta, Y, 1, \alpha_s(Q_0), \rho)\exp\left[-E_{\gamma\beta}^{(\mathrm{f})}\int_{Q_0}^{\mu}\frac{d\mu'}{\mu'}\left(\frac{\beta_0}{2\pi}\alpha_s(\mu')\right)\right]
$$

$$
E_{\gamma\beta}^{(\mathrm{f})}(\Delta\eta, Y, \rho) = \frac{2}{\beta_0}[\lambda_\gamma^{(\mathrm{f},1)*}(\Delta\eta, Y, \rho) + \lambda_\beta^{(\mathrm{f},1)}(\Delta\eta, Y, \rho)]
$$

  - Could also do numerically
    - [Banfi, Salam, Zanderighi (2002), Almeida, Sterman, Vogelsang (2008)]

RESUMMATION

- The cross section for heavy quark production with soft gluon emission of energy up to Q_0 becomes

$$\frac{d\hat{\sigma}^{(\mathrm{f})}_{G/Z'}}{d\Delta\eta} = \sum_{\beta,\gamma} (H^{(\mathrm{f},LO)}_{G/Z'})_{\beta\gamma}(M, m_Q, \Delta\eta, \alpha_s(p_T))\, S^{(\mathrm{f},0)}_{\gamma\beta} \left[\frac{\ln\left(\frac{Q_0}{\Lambda}\right)}{\ln\left(\frac{p_T}{\Lambda}\right)}\right]^{E^{(\mathrm{f})}_{\gamma\beta}}$$

M : Resonance mass

Q_0 : Energy of radiation into the gap

- Model dependence is encapsulated in a hard function.
- H and S are connected only through color indices.

GAP FRACTION

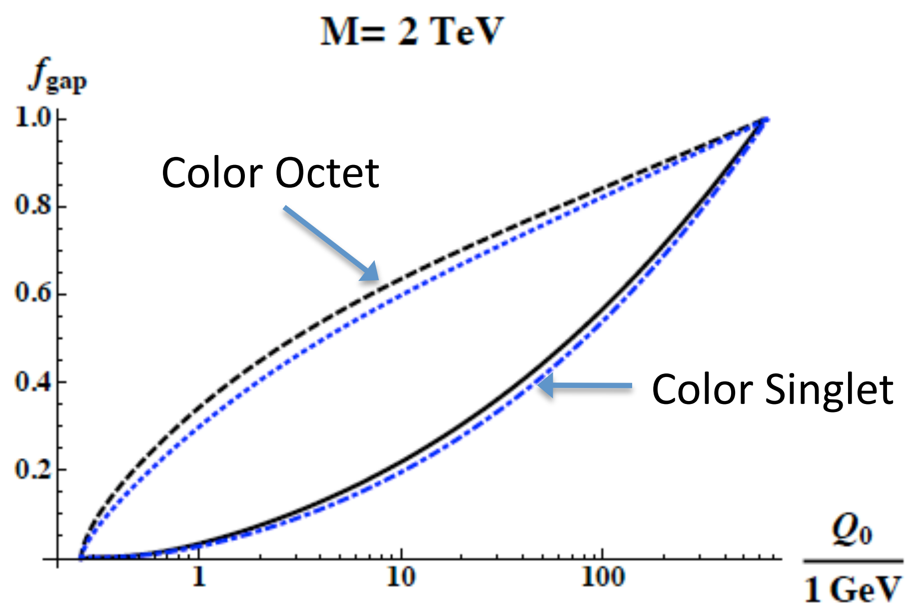
- The ratio of the number of events for heavy quark production with a specified rapidity gap to the total number of pair production events

$$f_{gap}^{(LO)}(M, Q_0) = \frac{\frac{d\hat{\sigma}^{(\mathrm{f})}(M,Q_0)}{d\Delta\eta}}{\frac{d\hat{\sigma}^{(\mathrm{f},\ \mathrm{LO})}(M,M)}{d\Delta\eta}}$$

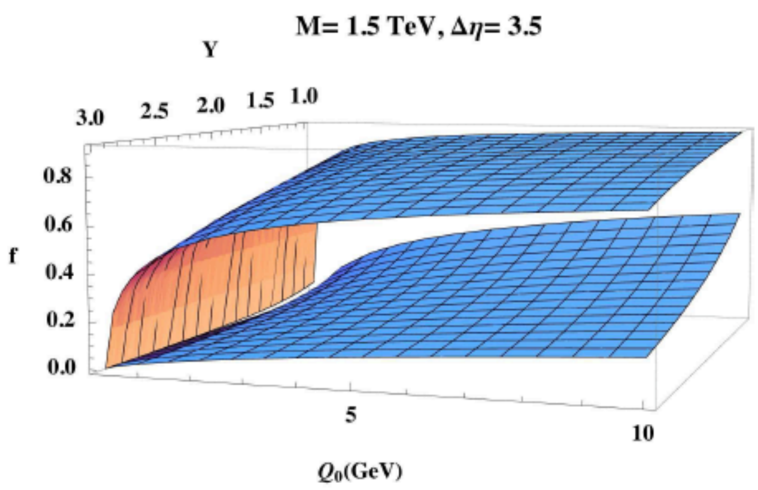
$$\frac{d\hat{\sigma}^{(\mathrm{f},LO)}}{d\Delta\eta} = \sum_{\beta,\gamma} H_{\beta\gamma}^{(\mathrm{f},LO)}(M, m_Q, \Delta\eta, \alpha_s(p_T))\, S_{\gamma\beta}^{(\mathrm{f},0)}$$

We don’t need to know about the strength of couplings of new sectors to a quark pair !

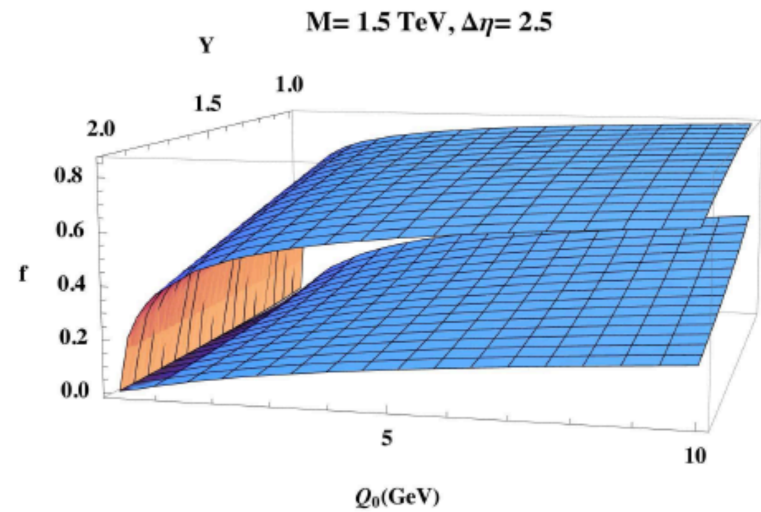
GAP FRACTION



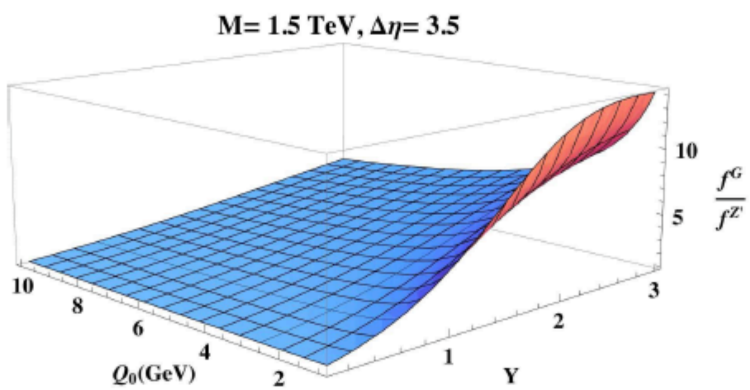
- More radiation for singlet than for octet resonances into the gap.

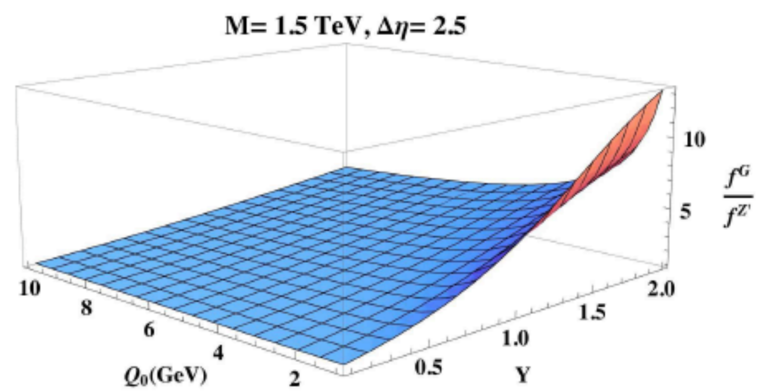

M= 1.5 TeV, Δη= 3.5
Y
3.0
2.5
2.0
1.5
1.0
0.8
0.6
f
0.4
0.2
0.0
5
10
Q0(GeV)

(a)


M= 1.5 TeV, Δη= 2.5
Y
2.0
1.5
1.0
0.8
0.6
f
0.4
0.2
0.0
5
10
Q0(GeV)

(b)


M= 1.5 TeV, Δη= 3.5
10
5
fG/fZ'
10
8
6
4
2
Q0(GeV)
1
2
3
Y


M= 1.5 TeV, Δη= 2.5
10
5
fG/fZ'
10
8
6
4
2
Q0(GeV)
0.5
1.0
1.5
2.0
Y

SUMMARY

- Color flow is important information for BSM search
- We have shown that it is possible to determine the color SU(3) representation of resonance from NP
- More radiation for singlet than for octet resonances
- We can use any fixed region of rapidity and azimuthal angle rather than the simple rapidity gap
- We can use color flow information to distinguish signals from backgrounds at the LHC
  - Higgs or Color singlet object-> jets vs QCD jets

From analytical LL resummed result

(left - work in progress, pp>HZ>b b-bar Z)

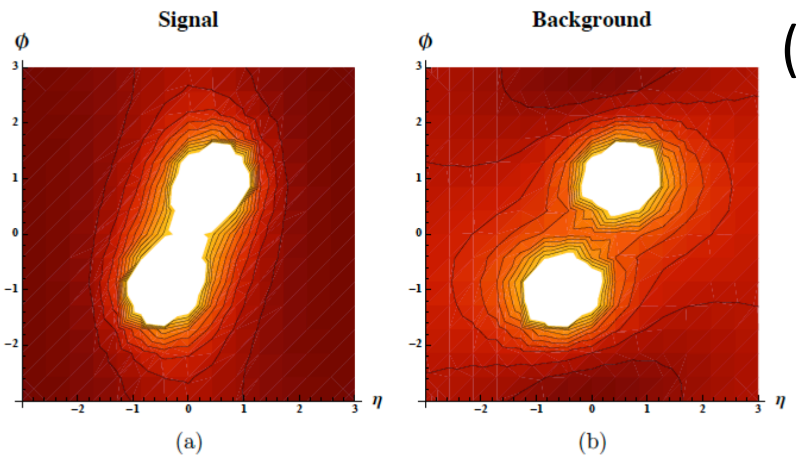
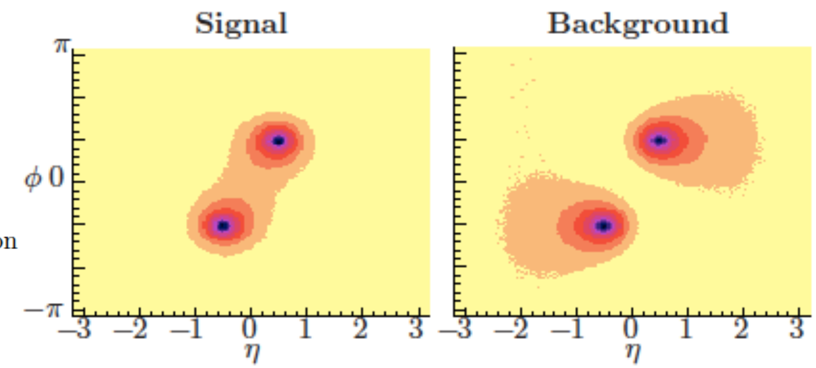


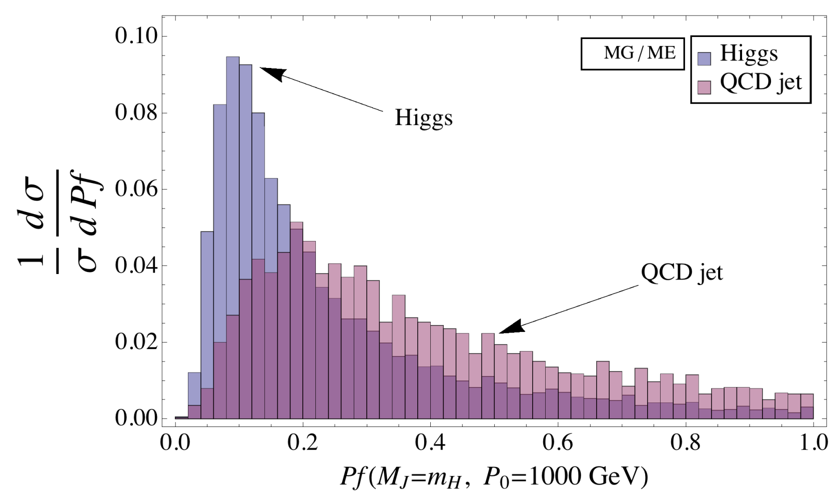
Figure 1: Density plots for $\frac{d\sigma}{dQ_c d\eta d\phi}$ from Higgs vs from background process with gluon of Q_c into (η, ϕ). Brighter color, more probability to emit gluon into the area.

Comparing to the result from MC (below) [Gallicchio, Schwartz (2010)]

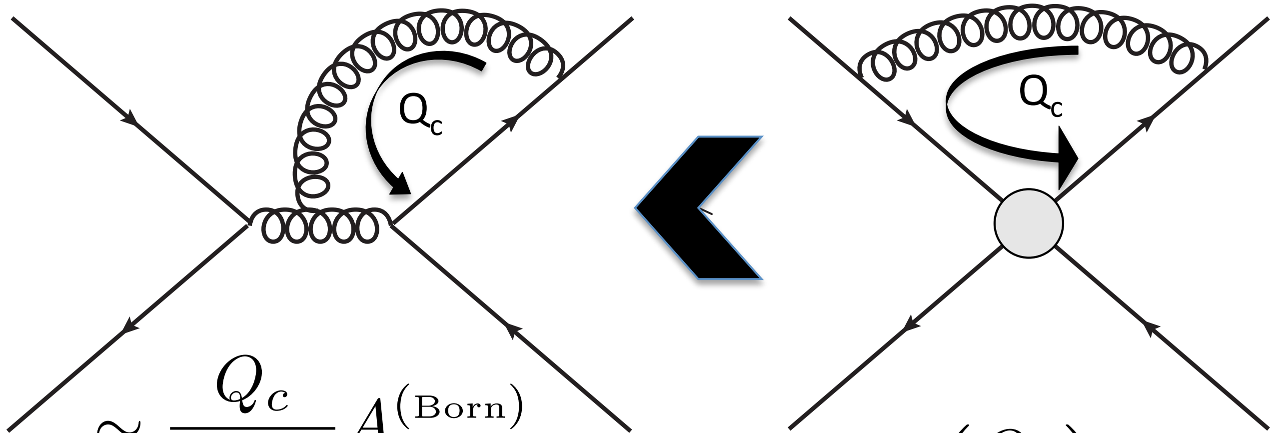


From MC simulations for Planar flow,

Almeida, Lee, Perez, Sterman, IS, Arxiv:1006.2035



- In this work, we consider resonance widths $\Gamma >$ O(GeV) for TeV resonance masses.



$$\sim \frac{Q_c}{\sqrt{\Gamma M}} A^{(\mathrm{Born})} \qquad\qquad \sim \ln\left(\frac{Q_c}{\Lambda}\right) A^{(\mathrm{Born})}$$

$$\begin{aligned} Q_c &< \sqrt{\Gamma M} \\ Q_c &\simeq \mathcal{O}(\mathrm{GeV}) \\ M &\simeq \mathcal{O}(\mathrm{TeV}) \end{aligned}$$

Q_c : Energy measure of radiation

$$\Gamma \simeq \frac{M}{6} \text{ for KK gluon in the RS models satisfying EW precision test [PRD, Agashe, et al (2008)]}$$

$$\Gamma \simeq \frac{M}{10} \text{ for axigluons and universal colorons [PRD, Choudhury, et al (2007)]}$$